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A Procedure for Scheduling Inventory of an Industry by Merging Forecasting and Linear Programming

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Abstract

In this paper, we develop a mathematical model combining forecasting and linear programming for a business organization of Bangladesh to calculate optimum order quantity and inventory cost. We test the model using raw data of the demand for the raw materials and spare inventory for the industry and find out minimum total inventory cost along with ordering cost and inventory holding cost. The developed model make a match between the forecasted demand of raw materials and spare inventory and the minimum total cost of inventory. Finally comparing minimum cost, we observe that our estimated appropriate forecasting method gives optimal inventory cost.

Keywords: Forecast; Inventory management; Linear programming; Error measurement.

I. Introduction

To achieve reliable customer service with right goods, in sufficient quantities, in the right place, at the right time with minimum cost every business organization has to set policies and controls to monitor levels of inventory and determine what levels should be maintained. On the other hand, the inventory management depends on reliable forecasting for the demands of finished products and supply of raw materials. On the basis of the number of purchasing times, the inventory system is classified into single and multiple-period inventory systems. In single period inventory system the firm decides to purchase just a one-time to cover a fixed period of time and the item will not be reordered. In multi period inventory systems an item will be available on an ongoing basis throughout the planning horizon. Multi period inventory systems is divided in to two parts known as fixed–order (economic order) quantity models (EOQ) which are event based models and fixed–time period models which are time based models. Every production department, be it a factory, a workshop or an engineering department, has to handle materials with the objective of achieving maximum customer's service and minimum cost. Forecasting and optimization have traditionally been approached as two distinct, sequential components of inventory management. The random demand is first estimated using historical data, then this forecast is used as input to the optimization module. In particular, time series analysis is a tool for developing mathematical models by assessing past data. These models are used in making forecasting decisions where the goal is to predict the next period's observation as precisely as possible.

There are many research articles which introduced different forecasting approaches to generate forecast of demand for inventory control purpose. In this section, some relevant articles are reviewed.

Kerkkanen *et al*.[9] observed that the demand forecasts based on historical demand are usually quite accurate for relatively smooth and continuous, pattern. Chandra and Grabis[4] studied the impact of the forecasting method selection on inventory performance. K. Skouri & S. Papachristos[14] studied on a continuous review of inventory models. They considered deterioration, inventory holding cost, shortage cost and the replenishment cost which is linear dependent on the lot size. Gutièrrez *et al*[7] analyzed the storage capacity of dynamic lot size problem. Computational results were reported for randomly generated problems. Forecasting and errors reduction was discussed in[1, 6, 10]. Impact of interaction between demand forecasting and inventory control was presented in[4, 16, 17]. Evaluation of forecasting methods for different industries were analyzed in[2, 5, 12, 15].

The objective of this paper is to identify an appropriate forecasting method and a linear programming (LP) model which can help the manager to achieve target of reorder quantity that is to maintain the suitable inventory level and which will give the optimal inventory cost. Here, we develop a mathematical model combining forecasting and LP for a business organization of Bangladesh which make a match between the forecasting of demand and supply and the minimum total cost of inventory. We test the model using raw data of the demand for the raw materials and spare inventory for the industry and find out minimum total inventory cost.

The rest of the paper is organized as follows. In Section II, existing forecasting techniques are discussed. In Section III we develop the linear programming (LP) model for optimal inventory cost calculation. In Section IV, we analyze a real life problem from the data of a Bangladeshi company. Section V concludes the paper.

II. Existing Forecasting Techniques

To develop a mathematical model collaborated with product manufacturing through which we can find out optimal inventory cost first we have to use forecasting methods. For forecasting the following techniques are used.

Single Moving Average (SMA): The following formula is used to compute the forecast

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$$F_t = MA_n = \frac{\sum_{i=1}^{n} A_{t-i}}{n} = \frac{A_{t-n} + \cdots + A_{t-2} + A_{t-1}}{n}$$

where,

$i =$ An index that corresponds to time period

$n =$ Number of periods in the single moving average

$A_{t-1} =$ Actual value in period $t-1$

$F_t =$ Forecast for time period t

$MA_n = n$ Period moving average

Double Moving Average (DMA): Five equations are used in the double moving average:

Simple moving average of n periods:

$$M_t = F_{t+1} = \frac{Y_t + Y_{t-1} + Y_{t-2} + \cdots + Y_{t-n+1}}{n}$$

Double moving average of n periods:

$$M^*{}_t = \frac{M_t + M_{t-1} + M_{t-2} + \cdots + M_{t-n+1}}{n}$$

Level at time t:

$$a_t = 2M_t - M^*{}_t$$

Slope at time t:

$$b_t = \frac{2(M_t - M^*{}_t)}{n-1}$$

Forecast at time $t+m$:

$$F_{t+m} = A_t + m * B_t$$

where,

$n =$ The number of period in the double moving average

Y_t= The actual series value at time period t

$m =$ Number of period ahead to be forecast

Weighted Moving Average (WMA): The weighted moving average can be computed by the following formula

$$F_t = w_n A_{t-n} + w_{n-1} A_{t-(n-1)} + w_{n-2} A_{t-(n-2)} + \cdots w_2 A_{t-2} + w_1 A_{t-1}$$

Satisfying $\sum_{i=1}^{n} w_i = 1$ and $w_1 \neq w_2 \neq w_3 \ldots . \neq w_n$

where,

F_t =Forecast for time period t

and $w_n =$ Weight for the period n.

Single Exponential Smoothing (SEM): The following formula is used to compute the forecast

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

where,

$F_t =$ The new smoothing value or the forecast value for time period t

F_{t-1} = Forecast for the previous time period.

$\alpha =$ The smoothing constant $(0 < \alpha < 1)$

A_{t-1} = Actual value in the previous period

Double Exponential Smoothing (DES): Five equations are employed:

Simple exponential smoothing forecast:

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

Double exponential smoothing forecast:

$$F^*{}_t = F^*{}_{t-1} + \alpha(F_{t-1} - F^*{}_{t-1})$$

Level at time t:

$$L_t = 2F_t - F^*{}_t$$

Slope at time t:

$$T_t = \frac{\alpha(F_t - F^*{}_t)}{1-\alpha}$$

Forecast at time $t+m$:

$$F_{t+m} = L_t + m * T_t$$

where,

F_t =The exponentially smoothed value of A_t at time t

$F^*{}_t$ =The double exponentially smoothed value of A_t at time t

F_{t+k} =The new smoothed value or the forecast value at the end of t period

α =The smoothing constant $(0 < \alpha < 1)$

A_{t-1} = Actual value in the previous period

Holt's Method (HM): At the end of the t period Holt method estimate the base level L_t and the pre-period trend T_t as follows

$$L_t = \alpha A_t + (1-\alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1}$$

where, α and β are smoothing constants and range is $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$

Then the forecast $F_{t,k}$ for A_{t+k} period is made at the end of t period by the formula

$$F_{t,k} = L_t + kT_t$$

III. Development of Linear Programming Model in Inventory Management

In this section, we develop a mathematical model through which we find out optimal inventory cost using the forecasted demand of raw materials and spare inventory.

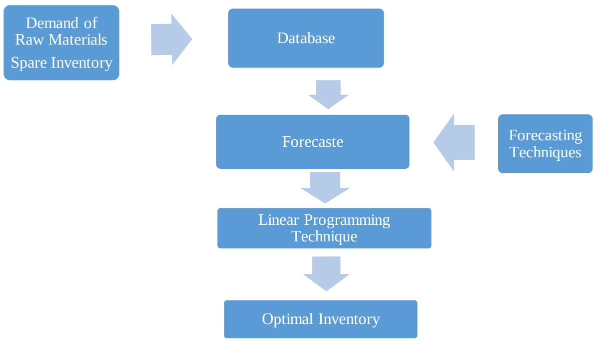


Fig. 1. Flowchart of the Research Model

d_t = The demand of raw materials in period t ($t = 1,2 \ldots . T$)
S = The warehouse storage capacity in item units
x_t = The order quantity in period t
I_t =The inventory level at the end of period t
$c_t x_t$ =The ordering cost incurred in period t
$h_t I_t$ =The holding cost in period t
where, x_t($t = 1,2 \ldots T$) represent the decision variables.

Figure 1 represents the flowchart of the research model. Firstly, demand of raw materials and spare inventory are collected. Collected database are analyzed through forecasting techniques. For conveniences the nearest integer values are considered. Then LP model are used for analyzed data to get the optimal inventory. To developed linear programming model the follows notations are used

Taking into account the above notation the LP equation formulated here is as follows:

$$\text{Min} \quad z = \sum_{t=1}^{T} (c_t x_t + h_t I_t)$$

Subject to,

$$I_{t-1} + x_t - d_t = I_t$$

$$I_{t-1} + x_t \leq S \tag{1}$$

$$x_t, I_t \in \mathbb{Z}^+ \qquad t = 1,2 \ldots \ldots T$$

The first set of constraint in (1) represents the well-known material balance equations which determine the inventory levels from the previous decisions. The second set of constraints indicates that the sum of the inventory level and the order quantity in period t must be smaller than or equal to the storage capacity in items units, and it avoids the order quantity exceeding the free storage capacity. The last set of constraints forces the order quantities and the inventory levels to be nonnegative integers. Assumed that the inventory level at the beginning of the first period and at the end of the final period are not zero. Then the first and second set of constraints in (1) can be used to obtain the following new constraints.

$$d_t + I_t \leq S$$

That is, $I_t \leq S - d_t \qquad t = 1,2, \ldots \ldots T$

Therefore the problem is reformulated as shown below.

$$\text{Min} \quad z = \sum_{t=1}^{T} (c_t x_t + h_t I_t)$$

Subject to,

$$I_0 \neq I_t \neq 0$$

$$I_{t-1} + x_t - d_t = I_t,$$

$$0 \leq I_t \leq S - d_t \tag{2}$$

$$x_t, I_t \in \mathbb{Z}^+ \qquad t = 1,2 \ldots \ldots T$$

Since the inventory level must be nonnegative and considering the second constraint of (2), the problem reaches a feasible solution whenever $d_t \leq S$, $t = 1,2 \ldots \ldots T$ .The warehouse must contain d_t units of item at the beginning of period t, but this would be impossible since the inventory level would exceed the capacity S.

IV. Detail Experimentation for a Typical Instance

Example 1: The industry considered in this research has two sections named as Warehouse-1 and Warehouse-2 where inventories are stored. The capacity of store area for each month of the industry is 4500 units. The industry needs approximately 600-1000 and 1500-2000 units' order of quantities for Warehouse-1 and Warehouse-2 in each month respectively. The actual demand of quantities and spare inventory with ordering cost for different warehouses and holding cost of the industry for one year period is:

Table 1. Actual demand of raw materials and spare inventory for one year

Month	Demand (In total)	Ordering Cost (Per unit)		Spare Inventory (In total)	Holding Cost (Per unit)
		Warehouse 1	Warehouse 2		
Jan	2370	15	17	1340	4
Feb	2400	22	20	1430	8
Mar	2410	17	16	1440	6
Apr	2370	23	21	1300	6
May	2450	25	22	1350	9
Jun	2500	16	19	1250	7
Jul	2430	13	15	1400	8
Aug	2470	18	17	1370	4
Sep	2560	19	21	1430	3
Oct	2520	16	18	1260	5
Nov	2550	17	19	1320	6
Dec	2540	18	20	1390	7

Forecasts obtained from different methods: The table 2 given in the below represents the forecasts of demand of raw materials and spare inventory obtained from forecasting models such as SMA, DMA, SES, DES, WMA and HM. Since demand of materials and quantity of spare inventory must be in integer form, we have considered here approximately nearest integer of the forecasted value for the different forecasting methods in Table 2.

Table 2. Forecast values in nearest integer form

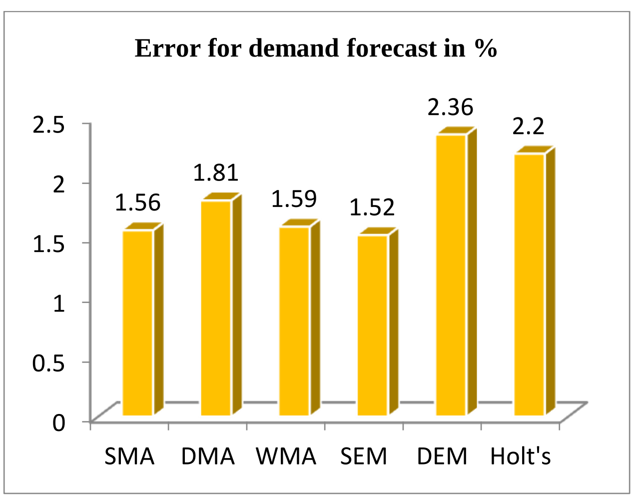
Month	Demand of Materials							Spare Inventory						
	Actual	SMA	DMA	WMA	SES	DES	Holt	Actual	SMA	DMA	WMA	SES	DES	Holt
Jan	2370				2400		2556	1340				1430		1395
Feb	2400				2385	2430	2450	1430				1385	1520	1364
Mar	2410				2393	2470	2405	1440				1408	1340	1403
Apr	2370	2393		2399	2401	2400	2388	1300	1403		1417	1424	1430	1433
May	2450	2393		2388	2386	2423	2357	1350	1390		1368	1362	1468	1358
Jun	2500	2410	2432	2418	2418	2365	2395	1250	1363	1318	1353	1356	1260	1345
Jul	2430	2440	2491	2459	2459	2472	2455	1400	1300	1198	1290	1303	1293	1274
Aug	2470	2460	2507	2455	2445	2568	2446	1370	1333	1336	1345	1352	1166	1332
Sep	2560	2467	2487	2464	2457	2470	2466	1430	1340	1371	1355	1361	1380	1352
Oct	2520	2487	2518	2507	2509	2496	2534	1260	1400	1484	1406	1395	1393	1404
Nov	2550	2517	2570	2522	2514	2631	2546	1320	1353	1331	1333	1328	1481	1323
Dec	2540	2543	2599	2543	2532	2587	2568	1390	1337	1283	1324	1324	1235	1312

Error Measurement. Many common techniques such as Mean absolute deviation (MAD), Mean squired error (MSE), Mean absolute percentage error (MAPE) and Root mean squired error (RMSE) are adopted to assess the accuracy of the forecasting methods. Smaller forecasted error gives the more the accuracy of forecasting methods. Among the all technique MAPE is one of the popular foresting measurement tools. The formula for MAPE is as bellow,

$$MAPE = \frac{\sum \frac{|A_t - F_t|}{A_t} \times 100\%}{n}$$

Using the formula calculated error in percentage for different forecasting techniques are given below in Figure 2.

Figure 2 shows the error percentages for different forecasting methods. It is clear that SES technique is the most appropriate technique for forecasting both for the demand of raw materials and sphere inventories since it gives the lowest error.



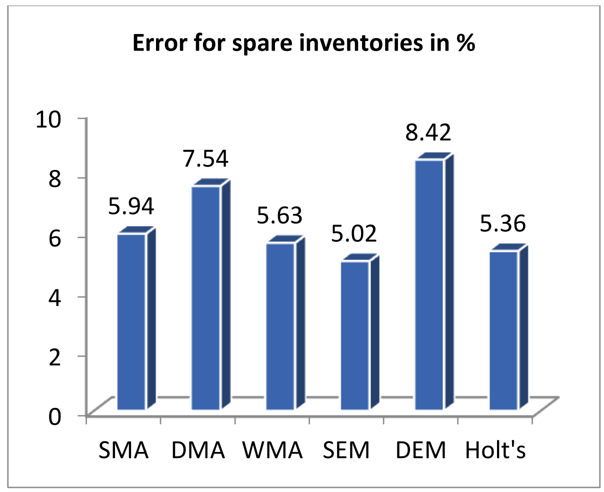


Fig. 2. Error Percentage in MAPE

Inventory Cost Calculation: For the numerical formulation of the case study of the industry considered here, we start the calculation of the equations from the beginning of the month and the demand of present month. The parameters for calculation are identified as in Table 3

Table 3. Variables and Parameter for Calculation

Time	-1	0	1	2
Demand	---	d_0	d_1	d_2
Warehouse 1	---	x_{10}	x_{11}	x_{12}
Warehouse 2	---	x_{20}	x_{21}	x_{22}
Spare Inventory	I_{-1}	I_0	I_1	I_2

Ordering cost for every month is different depending on their circumstances. The cost parameters for calculation are identified as in Table 4.

Table 4. Cost Parameters for Calculation

Time	0	1	2
Warehouse 1	c_{10}	c_{11}	c_{12}
Warehouse 2	c_{20}	c_{21}	c_{22}
Holding Cost	h_0	h_1	h_2

Here,

x_{10}, x_{11}, x_{11} = Order quantities for October, November and December month of Warehouse 1

x_{20}, x_{21}, x_{22} = Order quantities for October, November and December month of Warehouse 2.

I_{-1}, I_0, I_1, I_2 = Spare inventory at the end of September October, November and December.

d_0, d_1, d_2 = Demands of material for the October, November and December.

c_{10}, c_{11}, c_{12} = The ordering cost per unit for October, November and December of Warehouse 1.

c_{20}, c_{21}, c_{22} =The ordering cost per unit for October, November and December of Warehouse 2.

h_0, h_1, h_2 = The holding cost per unit for October, November and December.

Using the all above parameters the equation models that can solve problem of raw materials and inventory control of the industry can be summarized as bellow:

$$\text{Min } z = c_{10}x_{10} + c_{11}x_{11} + c_{12}x_{12} + c_{20}x_{20} + c_{21}x_{21} + c_{22}x_{22} + h_0I_0 + h_1I_1 + h_2I_2$$

Subject to,

$$x_{10} + x_{20} + I_{-1} = d_0 + I_0$$

$$x_{11} + x_{21} + I_0 = d_1 + I_1$$

$$x_{12} + x_{22} + I_1 = d_2 + I_2$$

$$x_{10} + x_{20} + I_{-1} \leq 4500$$

$$x_{11} + x_{21} + I_0 \leq 4500$$

$$x_{12} + x_{22} + I_1 \leq 4500$$

$$600 \leq x_{10}, x_{11}, x_{12} \leq 1000$$

$$1500 \leq x_{20}, x_{21}, x_{22} \leq 2000$$

$$x_i, I_i \in \mathbb{Z}^+$$

Using TORA Software and choosing Linear Programming we get optimal order quantities for every warehouse for the last three months using all forecasting methods. The results with ordering and holding costs are given as table 5 below:

We observe that, the SES method yields the lowest inventory cost as it was judged the most appropriate forecasting method for the industry.

Table 5. Inventory cost calculation for Two warehouses

Forecasting Method	x_{10}	x_{11}	x_{12}	x_{20}	x_{21}	x_{22}	Ordering Cost	Holding Cost	Inventory Cost
SMA	1000	970	1000	1547	1500	1527	137376	24477	161853
DMA	1000	917	1000	1631	1500	1551	138467	24387	162854
WMA	1000	949	1000	1558	1500	1534	137357	24296	161653
SES	1000	947	1000	1543	1500	1528	136933	24211	161144
DES	1000	1000	841	1509	1719	1500	137961	24496	162457
Holt's	1000	965	1000	1586	1500	1557	138593	24142	162735

Example 2. Let**,** The industry considered in typical example 1 uses three warehouses and it needs approximately 600-1000, 800-1200 and 1000-1500units' order of quantities for Warehouse-1, Warehouse-2 and Warehouse-3 in each month respectively. Let, $\boldsymbol{x_{30}, x_{31}, x_{32}}$ be the order quantities and $\boldsymbol{c_{30}, c_{31}, c_{32}}$ be the ordering cost per unit for Warehouse-3 for the month October, November and December. We consider the ordering cost and holding cost per unit as follows.

$$\begin{array}{lll} c_{10} = 16 & c_{11} = 18 & c_{12} = 25 \\ c_{20} = 15 & c_{21} = 22 & c_{22} = 17 \\ c_{30} = 21 & c_{31} = 20 & c_{32} = 15 \\ h_0 = 8 & h_1 = 7 & h_2 = 5 \end{array}$$

Solving all LP problems, we get the output as summarized in Table 6.

Table 6. Inventory cost calculation for three warehouses

Forecasting Methods	SMA	DMA	WMA	SES	DES	HM
x_{10}	600	600	600	600	600	600
x_{11}	670	617	649	647	641	717
x_{12}	600	600	600	600	600	600
x_{20}	947	1031	958	943	909	1006
x_{21}	800	800	800	800	800	800
x_{22}	800	800	800	800	800	800
x_{30}	1000	1000	1000	1000	1000	1000
x_{31}	1000	1000	1000	1000	1000	1000
x_{32}	1127	1151	1134	1128	1041	1157
Ordering Cost	139970	140636	139862	139511	142592	142151
Holding Cost	27356	27604	27199	27076	27686	27053
Inventory Cost	167326	168240	167061	166587	170278	169204

From Table 6, it is also clear that the SES methods yields the minimum inventory cost. Therefore, we recommend that, the manager of the industry will first forecast for raw materials and spare inventory by SES method and then will optimize the inventory cost by applying the developed LP.

V. Conclusion

The aim of this paper was to develop a mathematical model combining forecasting and linear programming for a business organization of Bangladesh. For this, we first applied forecasting models such as single-moving average (SMA), double-moving average (DMA), single-exponential smoothing (SES), double-exponential smoothing (DES), weighted moving average (WMA) and Holt's method (HM) to forecast the demand of raw materials and spare inventory. Using the error measurement techniques, we observed that SES produced the most reliable forecasts. We then developed an LP model to minimize the inventory cost and calculating the optimal order quantity. We tested the model using raw data of the demand for the raw materials and spare inventory for the industry and find out minimum total inventory cost along with ordering

cost and inventory holding cost and observed that the minimum inventory cost was yielded by SES.

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